

Pecking order as a dynamic leverage theory

C.N. BAGLEY¹, D.K. GHOSH² and U. YAARI³

¹ *Stochastic Software Systems, Bala Cynwyd, USA*

² *St. John's University, 300 Howard Avenue, Staten Island, New York 10301, USA*

³ *School of Business, Rutgers University, Camden, New Jersey 08102, USA*

Static tradeoff theories, which do not explicitly treat the impact of transaction costs, do not explain the policy of asymmetry between frequent small debt transactions and infrequent large equity transactions. Nor do these theories explain why the debt ratio is allowed to wander a considerable distance from its alleged static optimum, or how much of a distance should be tolerated. We offer a class of diffusion models that mimic this behaviour in a stochastic-dynamic framework and are designed to optimize a financing strategy using any static tradeoff theory as input. The models developed reveal the determinants of the size and frequency of equity transactions and the range of values over which leverage variations are tolerated in four generic scenarios. They also yield a new formulation of the cost of capital that recognizes stochastic transaction costs and a penalty for deviation from any static-optimal leverage. Our class of models augments the pecking order theory, provides a flexible quantitative framework for its implementation as a decision tool, and facilitates the formulation of additional hypotheses for its empirical validation. Symmetrically, our results show the importance of dynamic factors in designing and interpreting empirical tests of static tradeoff theories. The results presented have important implications for the role played by static tradeoff theories in a stochastic-dynamic framework. One such implication is that the static-optimal leverage has no direct effect on the firm's leverage policy in this setting. The target leverage for refinancing transactions is different from the static-optimal leverage, and the mean leverage is generally different from both. As a consequence, the latter cannot be used to estimate the former. Another implication is that even when the mean leverage equals the static optimum, mean reversion is not an optimal behaviour and therefore not a legitimate test for the existence of a static tradeoff in a dynamic context. Still another implication is that wide variations in leverage ratios cannot be interpreted as evidence of leverage indifference. It follows that the pecking order theory is consistent with static tradeoff theories and does not require the assumption of leverage indifference.

Keywords: financial leverage, dynamic debt policy, capital structure

1. INTRODUCTION

The dominant view of the firm's capital structure policy is that of a static tradeoff between debt and equity along a U-shaped capital cost function with a minimum defining the firm's optimal financial leverage. In its purest form, this view implies that the firm's leverage should not deviate from the optimum and

should change only when the cost function and its minimum change. The more practical version of the same view interprets frequent variations in the firm's capital structure as *small temporary excursions* from the optimum, caused by imperfect divisibility in the issuance or repurchase of claims. Since the optimal debt ratio is unobservable, researchers have sought evidence that the firm's fluctuating debt ratio reverts to a proxy of the optimal leverage – the mean debt ratio of the firm or its industry. Studies by Lev (1969), Taggart (1977), Bowen *et al.* (1982), Marsh (1982), Jalilvand and Harris (1984), Auerbach (1985) and Claggett (1992) report some evidence of leverage mean reversion, but recent empirical studies by Shyam-Sunder and Myers (1992) and Cai and Ghosh (1995) reject this hypothesis.

An apparently competing explanation for the observed fluctuations in the firm's debt ratio is provided by the pecking order theory advanced by Myers (1984a) and Myers and Majluf (1984). According to this theory, the firm's dividend policy is set to accommodate long-term growth opportunities, and is therefore sticky in the short term. Uncertain sources of cash available to meet frequently changing investment needs are selected based on their respective transaction costs, a concept broadly defined. Internal equity, which is free of such costs, is used first. When exhausted, it is followed by incremental borrowing characterized by low transaction costs. Beyond a certain limit, borrowing is followed by costly refinancing through the issuance of stock. A similar sequence of changes in the opposite direction is followed in response to an incremental excess of cash. This hypothesized hierarchy is strongly supported by findings of Shyam-Sunder and Myers (1992), Claggett (1992), and Cai and Ghosh (1995). Consistent with evidence reported below that incremental external equity but not debt has a large component of fixed transaction costs, the firm following this pecking order would allow its debt and debt ratio to fluctuate within *wide* limits. Stock would be issued or repurchased for the purpose of refinancing only infrequently, in lump sums, when those limits are reached. This scenario is further confirmed by Shyman-Sunder and Myers (1992), who show that frequent incremental changes observed in book debt ratios of US firms closely match contemporaneous changes in the demand for external funds. The latter changes are met almost entirely by incremental borrowing or debt retirement. The same study uses simulated data to show that mean reversion tests are strongly biased in favour of accepting the traditional hypothesis of static tradeoff.

Static tradeoff theories, which do not explicitly treat the impact of transaction costs, do not explain the policy of asymmetry between small and frequent debt transactions compared with large and infrequent equity transactions. Nor do these theories explain why the debt ratio is allowed to wander a considerable distance away from its alleged static optimum, or how much of a distance should be tolerated. These theories accommodate a stochastic demand for external funds *or* the presence of transaction costs, but not the *combination* of the two. For its part, the pecking order theory does not explain how a firm's static tradeoff would affect the pecking order behaviour. Nor does it specify the barriers that should be set by the firm to its fluctuating debt ratio, or the leverage adjustments to be made when those barriers are reached.

These issues are dealt with in this paper by a class of dynamic models that combine a static tradeoff (with leverage indifference as a limiting case), a stochastic demand for external funds, and the financing hierarchy of the pecking order theory induced by fixed equity transaction costs. The four stylized scenarios treated here are designed to focus the analysis on essential features of static tradeoff theories in a dynamic framework, and to allow in each case an analytical solution for an optimal inter-temporal financing strategy. The class of models offered can be turned into a normative tool when tailored to treat a specific realistic scenario of static tradeoff for which a numerical solution is sufficient.

Alternative dynamic models of the firm's leverage policy are offered by Fischer, Heinkel and Zechner (FHZ, 1989) and Mauer and Triantis (MT, 1994). Our differences with FHZ and MT, which are summarized in Appendix 1, concern the motivation, the theoretical framework, the scenarios modelled, the results and their interpretation. Their primary motivation is to provide a dynamic framework for a *specialized* static tradeoff theory combining the corporate interest tax shield of the US system with the cost of financial distress mitigated by the option to shut down production; our motivation is to create a quantitative structure accommodating side-by-side the pecking order theory and the family of theories of static tradeoff. Consistently, they use contingent-claims valuation models, and we prefer the (S,s) diffusion model which can assimilate the cost of capital function of various static theories. Based on their choice of a theoretical framework, they rely on the assumption that the firm's incremental investment is financed exclusively from shareholders' 'deep pocket': external funds in excess of cash reserves and current earnings come only from cuts in current dividends. While they disallow incremental borrowing between refinancing transactions, we follow the pecking order theory and supporting evidence in assuming that a sticky dividend policy causes incremental investment in excess of internal sources to be financed by incremental borrowing.

As a further difference from FHZ and MT, our class of models facilitates a closed-form solution for the optimal refinancing strategy under each of the four scenarios studied. New analytical expressions are derived for the leverage barriers set by the firm, the leverage readjustment target following encounters with the barriers, and the expected time interval between equity transactions. Relationships among these components of the optimal strategy are stated as elasticities that allow the formulation of sharper empirical hypotheses. An important by-product of the analytically defined optimal strategy is a new formula for the firm's cost of capital in a dynamic setting. This formula includes the contribution of stochastic transaction costs and the penalty associated with stochastic deviations from any static-optimal leverage, deviations that are not recognized in static models. Since there is a tradeoff between the penalty and transaction costs, the latter are determined endogenously.

Consistent with FHZ and MT, our analysis shows four concepts of 'optimal leverage' and matching leverage values that are associated with the firm's leverage policy in a dynamic setting. First, the cost function specifying the penalty paid for deviation from any static-optimal leverage is an input of the

leverage strategy. Second, the unique optimal return point,¹ i.e. the target debt ratio for leverage refinancing adjustments, is a component of that strategy. Third, the optimal mean leverage is a mere by-product of that strategy. Fourth, any leverage between the optimal upper and lower barriers is optimal since it does not justify intervention.

The distinct roles and generally different values of the static-optimal leverage, the optimal return point, and the mean leverage have important implications ignored by previous writers:

- (i) The pivotal role of the static-optimal leverage under static tradeoff theories vanishes under a dynamic policy induced by the presence of significant transaction costs.
- (ii) The observed mean leverage cannot be used as an estimate of the target ratio for leverage adjustments in a dynamic context, as often done in empirical studies.
- (iii) The observed mean leverage cannot be used as an estimate of the static-optimal leverage, as routinely done in earlier studies. A stable mean leverage does not prove the existence of a static-optimal leverage.

The following are broader implications of this study:

- (iv) Wide variations in leverage ratios, cross-sectionally and over time, cannot be cited as evidence of leverage indifference. The firm may be willing to pay a penalty to avoid higher refinancing costs.
- (v) Contrary to early assumptions by Myers (1984a) and Myers and Majluf (1984), the pecking order theory does not require the assumption of leverage indifference and is generally consistent with static tradeoff theories. One must not choose between the two approaches which complement one another rather than compete.
- (vi) The concept of 'target' debt ratio has a different meaning in the static and dynamic frameworks: it represents the static-optimal leverage in the former, and the optimal return point for refinancing transactions in the latter.
- (vii) In a dynamic setting, adjustments to the target ratio are made only as part of infrequent refinancing transactions executed when the debt ratio reaches its optimal upper or lower barrier. There is no partial reversion to the target ratio as long as the leverage lies between those barriers.
- (viii) Following from (vii), the existence of a static-optimal leverage is consistent with the absence of reversion to that optimum or to the mean leverage. Even when the mean equals the static optimum, mean reversion is not a legitimate test for the existence of a static tradeoff in a dynamic context.

¹ As shown numerically by Mauer and Triantis (1994) and proved by Bagley and Yaari (1996), the introduction of variable transaction costs would lead to a dual return point.

Despite differences between the theoretical frameworks, the assumptions and specific results, the general thrust of our findings is similar to that of FHZ and MT. This in itself is an important finding of this study which lends credibility to the results of all three studies.

The remainder of the paper is organized as follows. Section 2 summarizes the evidence of financing transaction costs, Section 3 describes the theoretical framework and basic assumptions shared by the four models derived, Section 4 provides the derivation of the models, Section 5 presents numerical examples, and Section 6 offers a summary and conclusions.

2. EVIDENCE OF TRANSACTION COSTS²

The empirical importance of the pecking order hierarchy hinges on the nature and significance of refinancing transaction costs. Evidence of economically significant stock issuance costs is reported in studies conducted on established US industrial firms traded on organized exchanges. Studying underwritten cash offerings, Asquith and Mullins (1986), Masulis and Korwar (1986), Mikkelsen and Partch (1986) and others cited by Smith (1986) report a permanent downward adjustment in the price of the stock held by existing shareholders, reaching nearly a third of the new money raised. The importance of a fixed cost component producing indivisibility is suggested by the absence of a significant cross-sectional relationship between issue size and the associated stock price adjustment. Given the strong association between firm size and issue size, dominant variable costs would cause instead a significant relationship between issue size and the negative price adjustment of existing stock. Direct evidence for the importance of fixed stock issuance costs is reported by Smith (1977) who shows substantial economies of scale in underwriters' compensation and direct administrative expenses – two important components of the overall issuance cost.

Contrary to issuance, where overall transaction costs measured by a stock price fall entail direct costs and a *related* negative information signal working in the same direction, the adverse price effect of transaction costs on repurchase may be masked by a larger *unrelated* positive price effect of a favourable signal. Empirical studies of the repurchase price premium by Masulis (1980), Dann (1981), and Vermaelen (1981) suggest that the sampled open-market purchases and tender offers are largely unrelated to leverage adjustments. These studies make no attempt to estimate the effect of transaction costs on the price premium. Measuring directly one category of transaction costs, Ferris *et al.* (1978) report that US companies repurchasing shares in lump sum via tender offer pay dealers a per-share soliciting fee averaging 6%, but reaching as high as 20% of the closing market price one week prior to the offer date. Significant fixed transaction costs on top of any variable soliciting costs would be entailed by the attention devoted by management to the typically large and infrequent tender offers. Since, according to evidence reported by Elton and Gruber (1968),

²This section and the following one (with the exception of assumption(e)) follow Bagley and Yaari (1996).

incremental open-market repurchases involve mainly variable transaction costs, the absolute and relative importance of fixed and variable costs may vary across firms depending on the repurchase method used.

The parallel evidence on debt indicates considerably smaller issuance costs, and by implication repayment costs. Eckbo (1986) reports an insignificant stock price adjustment to large debt offerings of US industrial firms. Mikkelsen and Partch (1986) report that the stock price adjustment associated with debt offerings is negative but less pronounced than that associated with stock offerings. Furthermore, private placements of debt and term loans have no significant effect on stock prices. This evidence is consistent with the presence of competing low-cost methods of raising debt privately, or publicly by 'shelf registration' under the SEC's Rule 415 of 1982.

3. THE THEORETICAL FRAMEWORK

Our objective is to design a theoretical framework in which various refinancing strategies can be explored. Our priority is to reach an analytical solution that would allow further analysis of the components of the optimal strategy. The following set of assumptions represents a balance between realism and mathematical tractability.

3.1 Assumptions

(a) Two classes of securities – debt and equity – are used by the firm to finance the stochastic demand for funds in excess of internal sources. The firm's dividend policy is set separately. Debt and equity are combined into a leverage index represented by a single state-control variable, x .

(b) The firm is in steady-state equilibrium. The combination of short-run and long-run equilibrium is necessary since our class of models does not specify the optimal path of policy change in reaction to a change of parameters.

(c) The firm's weighted average cost of capital (WACC) function is exogenous to our class of dynamic models. Static leverage theories enter these models by defining the appropriate fixed³ penalty function over the domain of x , a linear transformation of the WACC function. The penalty function is calculated by subtracting from the WACC at any leverage the minimum WACC and multiplying by the firm's value. By construction, the penalty function measures the *excess* annual dollar capital cost. If unique, the value of x for which the WACC function and the penalty function are at a minimum is referred to below as the *ideal* leverage, denoted by x_0 . The term 'optimal' is reserved hereafter for values derived by a dynamic strategy.

(d) The penalty function is assumed to incorporate any risk preference and be independent of the dynamic leverage strategies derived below. The latter assumption is consistent with results of MT showing that the firm's operating decisions are insensitive to its debt policy.

³ The models derived below can assimilate, with minor modifications, a stochastic ideal leverage with a fixed mean.

(e) The stochastic demand for external funds is met instantaneously and costlessly by incremental adjustments in borrowing. Infrequent refinancing transactions involving lump-sum offsetting changes in debt and equity are made without delay but at a fixed cost which is broadly interpreted to include all costs attributed to the transaction, excluding those that are merely coincidental (see Smith, 1986).⁴ The proceeds of stock issuance are used to retire debt, and stock repurchase is financed by borrowing. We ignore all transaction costs of incremental changes in the book value of debt, and – parallel with the Miller–Orr (1966, 1968) treatment of the demand for cash – any variable costs of equity transactions. Noting that variable costs alone would not generate the observed lumpy equity transactions, we opt for an assumption which is more easily modelled.⁵

(f) The state-control variable, x , which summarizes the capital structure of the firm, remains theoretically unspecified. In the present context, the correct specification of this leverage index is an empirical question. To be consistent with our models, this index must be monotonically increasing with the amount of debt, and decreasing with increases in the amount of equity. This index must also follow the diffusion process described below within the relevant range of the optimal control barriers.

(g) The leverage index state variable follows a Wiener process with constant drift, μ , and diffusion parameter, σ ,

$$dx = \mu dt + \sigma d\omega$$

The use of this process is compatible with the theoretical arguments of Myers (1984a) and the consistent evidence of studies of Stonehill *et al.* (1975), Marsh (1982), Titman and Wessels (1988), and Shyam-Sunder and Myers (1992), whereby the firm's leverage policy is dominated by the relationship between book rather than market values of debt and equity. The focus of our models on the transaction volatility of the book-based leverage is in unison with the pecking order theory.

3.2 The game plan

Our models make use of the (S,s) inventory control strategy formulated by Scarf (1960) and Iglehart (1963), as interpreted by Karlin and Taylor (1981). Under the scenario to be treated, the state variable triggers a fixed transaction cost at each encounter with the upper or lower barrier and generates an instantaneous penalty at all times while moving freely between the barriers. The firm's optimum control strategy calls for the positioning of the barriers and a single return point between them subject to the rule that the state variable be instantaneously readjusted to the return point each time it hits a barrier. The

⁴ The requirement of a lead time between the decision and execution of equity transactions could be readily assimilated in the models presented below if the costs of reversing the decision is prohibitive.

⁵ The alternative assumption that debt transactions have a fixed cost smaller than that of equity transactions would be inconsistent with the underlying continuous process. The introduction of variable transaction costs alongside the fixed costs would preclude an analytical solution but not change the nature of the results (Bagley and Yaari, 1996).

location of the barriers and the return point is selected so as to minimize the expected total periodic cost. In the case at hand, when the volatile leverage index hits the upper barrier, $x = b$, the firm makes a costly lump-sum readjustment in its capital structure to point $x = r$ by issuing stock and retiring bonds in the amount $b-r$. Similarly, when the leverage index hits the lower barrier, $x = a$, it is readjusted back to point $x = r$ by stock repurchase financed by borrowing in the amount $r-a$. Clearly $a \leq r \leq b$ and, for all x , $a \leq x \leq b$. As the leverage state variable driven by the process moves between the two barriers, it generates instantaneous costs that are a function of the value of the state variable along the penalty function. Note that the return point is allowed to differ from any ideal leverage inherent to the penalty function.

4. THE MODELS

The use of four specialized models instead of a single general one is essential for deriving closed-form solutions. The four stylized scenarios are designed to focus the analysis on salient features of static tradeoff theories in a dynamic setting.

Model 1 Endogenous barriers with a symmetric quadratic penalty function, symmetric transaction costs and no drift

4.1 Derivation

The assumed scenario of a symmetric penalty function around a unique ideal leverage with symmetric equity transaction costs and no drift serves two objectives. First, it shows that in the general case the choice of a refinancing strategy in the presence of fixed equity transaction costs involves the simultaneous optimization of three endogenous parameters: the upper barrier, the lower barrier, and the common return point. Second, it provides a benchmark for analysing the three types of asymmetry introduced subsequently in more realistic models.

The derivation starts with the intermediate value $s(x)$ subject to the condition $\mu = 0$

$$s(x) = e^{-\int \frac{2\mu(x)}{\sigma^2(x)} dx} = e^0 = 1 \quad (1)$$

The *scale function*, $S(x)$, is obtained from

$$S(x) = \int s(x) dx = x \quad (2)$$

The last function is used to obtain the *probability of hitting the upper barrier before the lower one*, $u(x)$, given that the process starts at an unspecified leverage x

$$u(x) = \frac{S(x) - S(a)}{S(b) - S(a)} = \frac{x - a}{b - a} \quad (3)$$

The *speed density*, $m(x)$, is calculated from equation (1)

$$m(x) = \frac{1}{\sigma^2 s(x)} = \frac{1}{\sigma^2} \tag{4}$$

This value is used to obtain the *expected time to reach either barrier* from any point x , $v(x)$, by substituting equations (2), (3), and (4) into the following integrals:

$$\begin{aligned} v(x) &= 2u(x) \int_x^b (S(b) - S(\xi))m(\xi)d\xi + 2(1 - u(x)) \int_a^x (S(\xi) - S(a))m(\xi)d\xi \\ &= \frac{(x - a)(b - x)}{\sigma^2} \end{aligned} \tag{5}$$

The *penalty function*, $g(x)$, used in this model is a symmetric U-shaped quadratic function centred on the ideal leverage, x_0 :

$$g(x) = q(x - x_0)^2 \quad q > 0 \tag{6}$$

This function, whose lowest value is $g(x_0) = 0$, measures the instantaneous penalty entailed by any deviation from the ideal leverage, x_0 . The rate at which the penalty increases as the leverage moves away from its ideal value is determined by q , which may be interpreted as a scale parameter proportional to the market value of the firm's assets. One virtue of this function is its consistency with the generic textbook WACC function that is increasing at an increasing rate with the distance from an ideal interior leverage. As a truncated Taylor polynomial, it may be viewed as a second-order approximation for the unknown true penalty function where simplicity is essential to obtain a closed-form solution. The unrealistic symmetry of this function makes Model I a useful benchmark for the other three models where symmetry is broken.

Next we derive the *expected penalty incurred up to the first encounter with either barrier*, $w(x)$, when starting at any point x

$$\begin{aligned} w(x) &= 2u(x) \int_x^b (S(b) - S(\xi))m(\xi)g(\xi)d\xi + 2(1 - u(x)) \int_a^x (S(\xi) - S(a))m(\xi)g(\xi)d\xi \\ &= \frac{q(x - a)(b - x)(a^2 + b^2 + ab + ax + bx + x^2 - 4ax_0 - 4bx_0 - 4xx_0 + 6x_0^2)}{6\sigma^2} \end{aligned} \tag{7}$$

The expected total cost incurred up to and including the first encounter with either barrier is the sum of the expected penalty, $w(x)$, and the fixed transaction cost, $k \geq 0$, incurred at the encounter point by issuing or repurchasing stock in lump sum.

The *objective function*, $z(x; a, b)$, is the sum of expected periodic penalty and transaction costs, calculated by multiplying the sum of the expected costs per encounter, $w(x) + k$, by the expected number of encounters per period, $1/v(x)$

$$z(x;a,b) = \frac{w(x) + k}{v(x)}$$

$$= \frac{k\sigma^2}{(b-x)(x-a)} + q \left(\frac{a^2 + ab + b^2 + ax + bx + x^2}{6} - \frac{2x_0(a+b+x)}{3} + x_0^2 \right) \quad (8)$$

This function represents the firm's *excess cost of capital*, measured in dollars over and above the amount consistent with the minimum WACC, stated in a general form with unspecified barriers and return point. It can be shown that adding a constant to the penalty function, $g(x)$, would result in adding the same constant to $z(x; a, b)$ without affecting the optimization. The firm's objective is to simultaneously select the optimal $a = a^*$, $b = b^*$ and $x = r^*$ that would minimize this expected periodic cost. This objective is simplified by recalling the symmetry of the process and expected costs with respect to the exogenous minimum point of the quadratic penalty function at $x = x_0$. As illustrated in Fig. 1, that symmetry implies that the optimal return point is at the ideal leverage, x_0 , half way between the optimal a^* and b^*

$$r^* = \frac{a^* + b^*}{2} = x_0 \quad (9)$$

Since the optimal barriers are symmetric with respect to x_0 , we simplify z by the substitutions $x = x_0$ and $2x_0 - b = a$

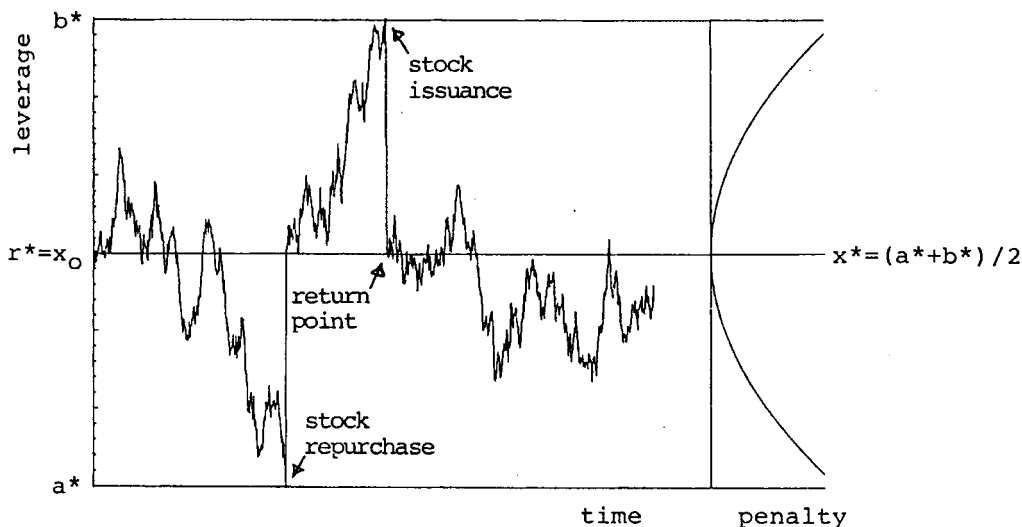


Fig. 1. Optimal leverage strategy in Model I: endogenous barriers, symmetric quadratic penalty function, symmetric transaction costs and no drift

$$z(a) = \frac{k\sigma^2}{(x_0 - a)^2} + \frac{(x_0 - a)^2 q}{6} \quad (8')$$

and then minimize $z(a)$ with respect to a to get the unique optimal value

$$a^* = x_0 - \left(\frac{6\sigma^2 k}{q}\right)^{\frac{1}{4}} \quad (10)$$

and symmetrically

$$b^* = x_0 - \left(\frac{6\sigma^2 k}{q}\right)^{\frac{1}{4}} \quad (11)$$

This result is consistent with intuition: given the firm's total asset value, the distance from the optimal return point (at the ideal leverage) to the optimal barriers is directly related to the unit transaction cost, k , and the leverage volatility, σ^2 , and inversely related to the penalty entailed by deviation from the ideal leverage, x_0 , a penalty that is proportional to parameter q . This parameter may be interpreted as a product $q = q_1 q_2$, where q_1 measures the per-dollar-asset steepness of the quadratic function and q_2 the total asset dollar value. An increase in q_2 proportionally decreases the relative size and effect of k so that the optimal distance between the barriers is directly related to q_1 and/or q_2 . (The extent of this effect is considered below.)⁶

The equality of the ideal leverage and the optimal return point stated in equation (9) is a consequence of the overall symmetry of this model, ensured by the assumed symmetry of the penalty function and equity transaction costs, and the absence of drift. As illustrated in Fig. 1 and demonstrated below, these assumptions further ensure that the optimal return point $r^* = x_0$ is also the optimal mean leverage over time, x^* .

4.2 Cost of capital

To derive the firm's *excess cost of capital*, the optimal r^* , a^* and b^* (given by, respectively, equations (9), (10) and (11)) are substituted in the objective function (8') where these parameters are unspecified

$$z^* = \left(\frac{2kq\sigma^2}{3}\right)^{1/2} \quad (12)$$

This function states the expected dollar periodic cost over and above the static minimum cost of capital. The traditional formulation accounts for non-stochastic investment-related flotation costs, assuming that the firm remains at the static optimal leverage, x_0 . *Under our formulation, the cost of capital includes flotation and repurchase costs in a stochastic form, as well as the cost of stochastic*

⁶ Despite fundamental differences between the two models, the signs of these effects are similar to those obtained by FHZ. This model is closer to that of FHZ than the remaining three models because of the assumed unique interior ideal leverage.

deviations from x_0 . Furthermore, our transaction costs are endogenous because of their tradeoff vis-à-vis the penalty paid for stochastic deviations from the ideal leverage.

4.3 Broad implications

Despite its simplicity and full symmetry, Model I has useful implications for the relationship between static tradeoff theories and the firm's behaviour in the presence of refinancing costs. First, wide variations observed in leverage ratios, cross-sectionally and over time, cannot be cited as evidence of leverage indifference. The firm may be willing to pay a penalty to avoid higher refinancing costs. Our quadratic penalty function has a unique ideal leverage, x_0 , while leverage volatility is tolerated between the barriers a^* and b^* . Second, the pecking order theory is consistent with static tradeoff theories and – contrary to the original assumption by its authors – does not require leverage indifference. One must not choose between the two approaches which complement each other rather than compete. Third, in a dynamic setting, adjustments to the target ratio, r^* , are made only as part of refinancing transactions executed when the debt ratio reaches its optimal upper or lower barrier. There is no reversion to the target ratio as long as the leverage lies between those barriers. Fourth, the existence of ideal leverage, x_0 , is consistent with the absence of reversion to that leverage or to the mean leverage, x^* . Even when the mean equals the ideal leverage, mean reversion is not a legitimate test for the existence of a static tradeoff in a dynamic setting.

4.4 Partial equilibrium relationships

The following additional results represent relationships implied by Model I. They form a basis for a positive theory describing how the real firm behaves and may be used to test this model. The derivation of these relationships is made possible by the closed-form solution for an optimal refinancing strategy.

The optimal a^* , b^* and r^* can be substituted in the general expressions for $u(x)$, $v(x)$ and $w(x)$ to obtain the optimal u^* , v^* and w^* . The optimal probability of hitting the upper barrier first is

$$u^* = \frac{1}{2} \quad (13)$$

which follows directly from the symmetry of the model [see equation (3)]. The optimal expected time between consecutive equity transactions

$$v^* = \frac{b^* - a^*}{2\sigma} = \left(\frac{6k}{q\sigma^2}\right)^{\frac{1}{2}} \quad (14)$$

is, as expected, related directly to the unit equity transaction cost and inversely to the penalty scale parameter and leverage volatility. The optimal expected penalty paid up to the first encounter with either barrier following a stock transaction is

$$w^* = k \quad (15)$$

which is independent of the penalty scale parameter and leverage volatility.

Having derived the optimal expected mean time between equity transactions, we can calculate x^* , the mean leverage associated with the optimal refinancing strategy. Although merely a by-product of the firm's leverage policy, this value has been the focus of extensive empirical research. The mean leverage is calculated in two steps: first, $w(x)$ in equation (7) is re-evaluated after substituting x for $g(x)$ and, respectively, the optimal parameters a^* , b^* and r^* for the unspecified a , b and x ; second, the result is divided by the optimal time interval, v^* ,

$$x^* = \frac{2u(x) \int_{r^*}^{b^*} (S(b^*) - S(\xi))m(\xi)\xi \, d\xi + 2(1 - u(x)) \int_{a^*}^{r^*} (S(\xi) - S(a^*))m(\xi)\xi \, d\xi}{v^*(x)} = r^* \tag{16}$$

In this special case, the dynamic optimal mean leverage is equal to the ideal leverage, x_0 , because of cost symmetry.

Testable partial-equilibrium conditions are provided by the following partial elasticities of v^* , b^* and z^* with respect to parameters σ^2 , k and q :

$\eta_{\text{row,column}}$	σ^2	k	q
v^*	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
b^*	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$
z^*	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

(17)

These elasticities have specific numerical values bearing the expected signs. When k and q are held constant, an increase in leverage volatility, σ^2 , causes a decrease in the expected time between equity transactions and an increase in the optimal distance between the upper and lower barriers (consistent with FHZ) and in the cost of capital. An increase in fixed equity transaction costs, k , causes an increase in the optimal distance between the barriers, the optimal time between equity transactions, and the cost of capital. Finally, an increase in firm size, reflected in an increase in q , diminishes the effect of k on v^* and b^* , having the reverse effects of an increased k . The effect of q on z^* has the same sign and magnitude as that of k : increases in q and k increase the dollar periodic cost of capital at a decreasing rate, indicating economy of scale.

Model II Endogenous upper barrier with an asymmetric quadratic penalty function, symmetric transaction costs and no drift

Real financing decisions are unlikely to be made under conditions of symmetry. To break the symmetry of Model I without giving up a closed-form solution, we now relax the assumption of a symmetric penalty function at the price of reducing the number of optimization parameters from three to two. This is accomplished by forcing the quadratic penalty function's minimum point through the origin and discarding the lower branch by setting an exogenous limit to the lower barrier at $a = 0$. As shown in Fig. 2, the resulting scenario is a

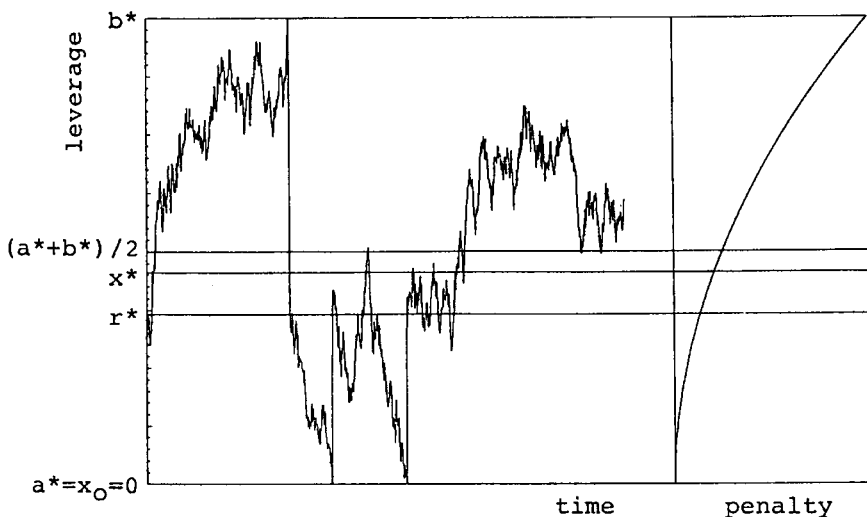


Fig. 2. Optimal leverage strategy in Model II: endogenous upper barrier, asymmetric quadratic penalty function, symmetric transaction costs and no drift ($0 = x_0 = a^* < r^* < x^* < (a^* + b^*)/2$)

monotonically increasing penalty over the relevant domain with an ideal leverage at the lower barrier, $x_0 = a = 0$. Our objective is simplified in this model to finding only the optimal upper barrier and return point. This scenario captures the essence of the static model advocated by Jensen and Meckling (1976), Myers (1984b), Townsend (1979), Gale and Hellwig (1985) and Bernanke and Gertler (1989, 1990), in which a monotonically increasing cost of financial distress is superimposed on a flat cost of capital function consistent with leverage indifference. (This hybrid textbook model is often combined with the interest tax shield of corporate borrowing to obtain an interior ideal leverage.)⁷

Following the derivation of Model I, we obtain the expressions for the intermediate values of Model II for $s(x)$, $S(x)$, $u(x)$, $m(x)$, $v(x)$, $g(x)$ and $w(x)$ as reported in Table 1. The resulting objective function

$$z(x) = \frac{k\sigma^2}{x(b-x)} + q \frac{b^2 + xb + x^2}{6} \quad (18)$$

is minimized simultaneously with respect to x and b to yield the optimal values

⁷ Like Miller and Orr's (1966, 1968) models for optimizing the firm's cash balance, Model II contains the combination of fixed transaction costs and a monotonic penalty function with a minimum at the natural barrier, $x_0 = a = 0$. Combined with our diffusion process, these conditions make it infinitely costly to linger at the ideal leverage, encouraging a one-step adjustment of x to a distant return point, $x = r > x_0 = a = 0$. This consequence is inherent to the underlying static model and could be avoided in a dynamic model allowing the (arbitrary) assumption of purely variable transaction costs (Bagley and Yaari, 1996).

$$r^* = \left(\left(\frac{4}{\sqrt{3}} - 2 \right) \frac{k\sigma^2}{q} \right)^{\frac{1}{4}} = 0.7458 \left(\frac{k\sigma^2}{q} \right)^{\frac{1}{4}} \tag{19}$$

$$b^* = \left(\left(\frac{2}{3} \right)^{\frac{1}{4}} + 6^{\frac{1}{4}} \right) \left((2\sqrt{3} - 3) \frac{k\sigma^2}{q} \right)^{\frac{1}{4}} = 2.038 \left(\frac{k\sigma^2}{q} \right)^{\frac{1}{4}} \tag{20}$$

where $r^*/b^* = 0.3660$. Unlike Model I, the optimal return point is closer to the exogenous lower barrier, $a^* = 0$, than to the optimal upper barrier due to a monotonically increasing penalty function.

The firm's excess cost of capital is $z^* = \sqrt{\frac{kq\sigma^2}{2\sqrt{3} - 3}}$ (21)

which corresponds to equation (12) in Model I. A summary listing of the objective function and key optimal values of Model II and subsequent models is provided in Table 2. The optimal probability of hitting the upper barrier first

$$u^* = \frac{1}{1 + \sqrt{3}} = 0.3660 \tag{22}$$

Table 1. Intermediate values of the four models

	Models I, II, III: Symmetric process	Model IV: Asymmetric process
$s(x)$	1	e^{xc}
$S(x)$	x	$\frac{e^{xc}}{c}$
$u(x)$	$\frac{x - a}{b - a}$	$\frac{e^{xc} - e^{ac}}{e^{bc} - e^{ac}}$
$m(x)$	σ^{-2}	$\frac{1}{2\sigma^2 e^{xc}}$
$v(x)$	$(x - a)(b - x)\sigma^{-2}$	$2 \frac{(x - a)e^{bc} + (b - x)e^{ac} + (a - b)e^{xc}}{c\sigma^2(e^{bc} - e^{ac})}$
$g(x)$	$q(x - x_0)^2$	0
$w(x)$	$\frac{(q(x - a)(b - x)(a + b + x) \times (a^2 + b^2 + ab + 6x_0^2 + x(x - 4x_0)))}{6\sigma^2}$	0

Model I takes the results reported above.
 Model II takes the results of Model I with the substitution $a = 0$ and $x_0 = 0$.
 Model III takes the results of Model I with the substitution $q = 0$.

Table 2. Objective function and optimal values

	Models I: Symmetric process	Model II: Asymmetric penalty costs
$z(x)$	$\frac{q(x_0 - a)^2}{6} + \frac{k\sigma^2}{(x_0 - a)^2}$	$\frac{k\sigma^2}{x(b-x)} + \frac{q(b^3 - x^3)}{6(b-x)}$
r^*	x_0	$0.7458\left(\frac{k\sigma^2}{q}\right)^{\frac{1}{4}}$
a^*	$x_0 - 1.565\left(\frac{k\sigma^2}{q}\right)^{\frac{1}{4}}$	Exogenous
b^*	$x_0 + 1.565\left(\frac{k\sigma^2}{q}\right)^{\frac{1}{4}}$	$2.038\left(\frac{k\sigma^2}{q}\right)^{\frac{1}{4}}$
z^*	$0.8165\sqrt{qk\sigma^2}$	$2.076\sqrt{qk\sigma^2}$
u^*	0.5000	0.3660
v^*	$2.449\sqrt{\frac{k}{q\sigma^2}}$	$0.9634\sqrt{\frac{k}{q\sigma^2}}$
w^*	k	k
x^*	r^*	$0.9278\left(\frac{k\sigma^2}{q}\right)^{\frac{1}{4}}$

is less than 0.5 due to the increasing penalty function. The optimal expected time between equity transactions is

$$v^* = \sqrt{2(2\sqrt{3} - 3)\frac{k}{q\sigma^2}} \quad (23)$$

which is about two-and-a-half times shorter than in Model I. The optimal expected penalty paid up to the first encounter with either barrier following a stock transaction is again

$$w^* = k \quad (24)$$

Despite the asymmetry of this model, the partial elasticities of z^* , u^* and v^* with respect to σ^2 , k and q are the same as those in Model I.

The derived optimal mean leverage is

$$x^* = \left(2\left(\frac{\sqrt{3} + 2}{\sqrt{3^3}}\right)^3 \frac{k\sigma^2}{q}\right)^{\frac{1}{4}} = 0.9278\left(\frac{k\sigma^2}{q}\right)^{\frac{1}{4}} \quad (25)$$

Table 2. Continued

	Model III: Asymmetric transaction costs	Model IV: Asymmetric process
$z(x)$	$\frac{\sigma^2(k_a(b-x) + k_b(x-a))}{(b-x)(x-a)(b-a)}$	$\frac{1}{2} \frac{ck\sigma^2(e^{bc} - e^{ac})}{(b-x)e^{ac} + (x-a)e^{bc} - (b-a)e^{xc}}$
r^*	$\frac{a\sqrt{k_b} + b\sqrt{k_a}}{\sqrt{k_a} + \sqrt{k_b}}$	$\frac{1}{c} \ln\left(\frac{e^{bc} - e^{ac}}{c(b-a)}\right)$
a^*	Exogenous	Exogenous
b^*	Exogenous	Exogenous
z^*	$\sigma^2\left(\frac{\sqrt{k_a} + \sqrt{k_b}}{b-a}\right)^2$	$\frac{kc^2\sigma^2(e^{bc} - e^{ac})}{2\left(e^{ac}(1+bc) - e^{bc}(1+ac) + (e^{bc} - e^{ac}) \ln\left(\frac{e^{ac} - e^{bc}}{c(a-b)}\right)\right)}$
u^*	$\frac{\sqrt{k_a}}{\sqrt{k_a} + \sqrt{k_b}}$	$\frac{e^{bc} + (ac - bc - 1)e^{ac}}{c(b-a)(e^{bc} - e^{ac})}$
v^*	$\frac{(b-a)^2\sqrt{k_a k_b}}{\sigma^2(\sqrt{k_a} + \sqrt{k_b})^2}$	$\frac{2}{c^2\sigma^2}\left(\ln\left(\frac{e^{ac} - e^{bc}}{c(a-b)}\right) - 1\right) - \frac{2(ae^{bc} - be^{ac})}{c\sigma^2(e^{bc} - e^{ac})}$
w^*	0	0
x^*	$\frac{(2a+b)\sqrt{k_b} + (a+2b)\sqrt{k_a}}{3(\sqrt{k_a} + \sqrt{k_b})}$	$\frac{\left(e^{ac}(1+cb)^2 - e^{bc}(1+ca)^2 + (e^{bc} - e^{ac}) \times \left(-2 - (a+b)c + \left(1 + \ln\left[\frac{e^{ac} - e^{bc}}{(a-b)c}\right]\right)^2\right)\right)}{2c\left(\frac{c(be^{ac} - ae^{bc})}{(a-b)c} + (e^{ac} - e^{bc})\left(1 - \ln\left[\frac{e^{ac} - e^{bc}}{(a-b)c}\right]\right)\right)}$

so that $x^*/b^* = 0.4553$ and $x^*/r^* = 1.2440$. As illustrated in Fig. 2, the optimal mean leverage falls above the optimal return point, but below the half-way mark between the lower and upper barriers, $0 = x_0 = a^* < r^* < x^* < (a^* + b^*)/2$.

The theoretical and empirical significance of these quantitative results are summarized by the following statements.

(a) The pivotal role of the static-optimal leverage under static tradeoff theories vanishes under a dynamic policy induced by the presence of significant refinancing costs. The ideal leverage, x_0 , does not enter the expressions for the optimal barriers or return point.

(b) Due to cost asymmetry, the dynamic-optimal mean leverage is distinct from the ideal leverage and the optimal return point.

(c) Due to their inequality, the observed mean leverage is a biased estimate of the target ratio for leverage adjustments in a dynamic context, a factor ignored in previous empirical studies.

(d) Due to their inequality, the observed mean leverage is a biased estimate of the ideal leverage, a factor overlooked in earlier studies. A stable mean leverage does not prove the existence of an ideal leverage.

(e) The concept of 'target' debt ratio has a different meaning in the static and dynamic frameworks: it represents the ideal leverage in the former, and the optimal return point for refinancing transactions in the latter.

(f) For the real firm operating in a dynamic setting, there are three distinct meanings for the term 'optimal leverage' (a fourth one is noted in the Introduction), and possibly three distinct respective values: an unobservable ideal leverage, x_0 , which would always be adhered to in the absence of transaction costs; an observable optimal return point, r^* , associated with the optimal refinancing strategy; and a mean leverage, x^* , that is an estimable by-product of that strategy.

Since from an empirical viewpoint an underlying cost symmetry would be the exception, the use of the mean as a proxy for the unobservable ideal leverage would introduce an unknown bias. This bias may be present in mean-reversion tests reported by Taggart (1977), Bowen *et al.* (1982), Marsh (1982), Jalilvand and Harris (1984), Auerbach (1985), Claggett (1992), Shyam-Sunder and Myers (1992), and Cai and Ghosh (1995). It may be supplemented by a second bias due to the difference between the estimated leverage mean and the optimal return point – the true target of occasional reversion from the barriers. Potential biases notwithstanding, a more fundamental problem with mean reversion tests is that reversion in the usual sense of frequent partial adjustments would be un-economic regardless of the central value selected. At the same time, any optimal infrequent jumps from the barriers to the optimal return point are likely to play a negligible role in such tests.

Model III Leverage indifference with exogenous barriers, asymmetric transaction costs and no drift

This one-parameter model assumes the setting of the Modigliani–Miller (1958) tax-free model and Miller's (1977) all-tax model, consistent with the pure version of the pecking order theory of Myers (1984a) and Myers and Majluf (1984). The absence of unique ideal leverage implies the absence of a penalty function and invites the imposition of exogenous limits to the upper and lower barriers to obviate the preference for unrealistic barriers at $a = -\infty$ and $b = \infty$.⁸ The mathematical simplification gained by assuming the absence of a penalty function allows us to introduce a new element of realism to the analysis by

⁸ The assumption of leverage indifference requires the additional assumption of no default risk. In the presence of default risk, leverage indifference can apply only to a newly established firm, since an unexpected change in the leverage of an established firm would destroy wealth or redistribute it among claimants – implying the presence of a penalty function and an ideal leverage.

breaking the symmetry in transaction costs. Let k_a be the fixed cost per stock repurchase and k_b the typically higher fixed cost per stock issuance. This modification accommodates but does not depend on Myers' (1984a) argument that a substantial cost of asymmetric information is absent from stock repurchase.

We again follow the derivation of Model I to obtain the intermediate results reported in Table 1. The objective function

$$\begin{aligned} z(x) &= \frac{u(x)k_b + (1 - u(x))k_a}{v(x)} \\ &= \frac{\sigma^2(k_a(b - x) + k_b(x - a))}{(b - x)(x - a)(b - a)} \end{aligned} \quad (26)$$

is minimized with respect to x to obtain

$$r^* = \frac{a\sqrt{k_b} + b\sqrt{k_a}}{\sqrt{k_a} + \sqrt{k_b}} \quad (27)$$

which is independent of σ^2 . In the special case $k_a = k_b$, this expression is reduced to $r^* = (a + b)/2$. If the transaction costs of stock repurchase are negligible ($k_a = 0$), this expression is further simplified to $r^* = a$, confirming the intuitive implication that the lower barrier becomes the optimal return point. Since $b > a$ and realistically $k_a < k_b$, it follows that $r^* < (a + b)/2$, which is the same effect achieved by an increasing penalty function in Model II.

Under this model, the firm's excess cost of capital is given by

$$z^* = \sigma^2 \left(\frac{\sqrt{k_a} + \sqrt{k_b}}{b - a} \right)^2 \quad (28)$$

which is reduced to $z^* = k(2\sigma/(b - a))^2$ under $k_a = k_b$. The optimal probability of hitting the upper barrier first is

$$u^* = \frac{\sqrt{k_a}}{\sqrt{k_a} + \sqrt{k_b}} \quad (29)$$

which is reduced to $u^* = 1/2$ under $k_a = k_b$. The optimal expected time between equity transactions is

$$v^* = \frac{(b - a)^2 \sqrt{k_a k_b}}{\sigma^2 (\sqrt{k_a} + \sqrt{k_b})^2} \quad (30)$$

which is simplified to $v^* = ((b - a)/2\sigma)^2$ under $k_a = k_b$. To allow comparison, the objective function and optimal values of Model III are displayed in Table 2 along with those of Models I and II and with those of Model IV derived below.

The implied optimal mean leverage is

$$x^* = \frac{(2a + b)\sqrt{k_b} + (a + 2b)\sqrt{k_a}}{3(\sqrt{k_a} + \sqrt{k_b})} \quad (31)$$

where clearly $x^* < r^*$ so that $x^* < (a + b)/2$. Like r^* , x^* is independent of σ^2 . In the special case $k_a = k_b$, the mean leverage becomes $x^* = (a + b)/2$. If transaction costs in stock repurchase, k_a , are negligible, this leverage assumes the simple expression $x^* = (2/3)a + (1/3)b$.

These results show that asymmetric transaction costs, like an asymmetric penalty function, lead to distinct mean leverage and return points, rendering the former a biased estimate of the latter. Furthermore, they show that well-defined mean leverage and return points are compatible with leverage indifference and the absence of an ideal leverage.

If we let $l_b = \sqrt{k_b/k_a}$, the partial elasticities of v^* , r^* and z^* in Model III are as follows:

$\eta_{\text{row,column}}$	σ^2	$\frac{k_b}{k_a}$	$b - a$	b	k_a
v^*	-1	$\frac{1 - l_b}{2(1 + l_b)}$	2	$\frac{2b}{b - a}$	$\frac{l_b - 1}{2(l_b + 1)}$
r^*	0	$\frac{-(b - a)l_b}{2(1 + l_b)(b + al_b)}$	-	$\frac{b}{b + al_b}$	$\frac{(b - a)l_b}{2(1 + l_b)(b + al_b)}$
z^*	1	$\frac{l_b}{1 + l_b}$	-2	$\frac{-2b}{b - a}$	$\frac{1}{1 + l_b}$

(32)

The elasticities with respect to a and k_b may be obtained by the transformations $a \Leftrightarrow b$, $k_b \Leftrightarrow k_a$, $l_a \Leftrightarrow 1/l_b$. Due to the absence of a penalty function in this model, the scale parameter, q , does not enter any of these elasticities, indicating the absence of economies of scale.

Despite differences in details, the results of this model are clearly consistent with those of Models I and II.

Model IV Leverage indifference with exogenous barriers, symmetric transaction costs and drift

In this model, symmetry is broken by augmenting the simplified diffusion process used above by a time-constant instantaneous drift, μ . To handle the resulting complications, this model is simplified by reinstating the assumption of symmetric transaction costs. The assumed absence of drift in Models I-III is consistent with the pecking order theory by which a firm in steady-state equilibrium would eliminate any drift by setting its dividend policy to ensure that any systematic growth is financed exclusively by retention and borrowing at the desired ratio. Nevertheless, temporary drift may last months or years in

the case of an investment which grows at a faster ($\mu > 0$) or slower ($\mu < 0$) rate than earnings.

To simplify the exposition, we take advantage of the fact that the model treats μ and σ^2 as constants to define the new constant $c = -2\mu/\sigma^2$. Following the derivations of Models I-III, we obtain the results displayed in Table 1.

The objective function

$$z(x) = \frac{ck\sigma^2(e^{bc} - e^{ac})}{2((b-x)e^{ac} + (x-a)e^{bc} - (b-a)e^{xc})} \quad (33)$$

is minimized with respect to x to obtain the return point

$$r^* = \frac{1}{c} \ln\left(\frac{e^{bc} - e^{ac}}{c(b-a)}\right) \quad (34)$$

To unburden the exposition, the optimal values z^* , u^* and v^* are reported only in Table 2. The cumbersome partial elasticities are not reported for lack of space.

The optimal mean leverage is

$$x^* = \frac{e^{ac}(1+cb)^2 - e^{bc}(1+ca)^2 + (e^{bc} - e^{ac})\left(-2 - (a+b)c + \left(1 + \ln\left[\frac{e^{ac} - e^{bc}}{(a-b)c}\right]\right)^2\right)}{2c\left(c(be^{ac} - ae^{bc}) + (e^{ac} - e^{bc})\left(1 - \ln\left[\frac{e^{ac} - e^{bc}}{(a-b)c}\right]\right)\right)} \quad (35)$$

where unlike Model III, the presence of drift causes r^* and x^* to depend on σ^2 through c . As expected, this dependence disappears at the limit as $\mu \rightarrow 0$ (that is $c \rightarrow 0$), causing $r^* = x^* = (a+b)/2$.

A general implication of these results is that $\mu \neq 0$ causes $r^* \neq x^*$, so that the optimal return point and expected leverage are again distinct. Like cost asymmetries in Models II and III, process asymmetry here renders the mean leverage a biased proxy for the optimal return point. As in Model III, these results further show that well-defined mean leverage and return points are consistent with leverage indifference and the absence of an ideal leverage.

5. NUMERICAL EXAMPLES

Using numerical examples based on analytical solutions, Tables 3 and 4 facilitate a comparative-static view of the four models applied to a firm of one dollar size and a penalty function, if any, defined over a leverage domain from zero to one. Due to the standardized firm size and leverage domain, the penalty function may be interpreted as a weighted-average excess cost of capital defined, for the sake of exposition, over the ratio debt/assets.

Model I. This model is displayed in Table 3 using a quadratic penalty function with a minimum value of zero at the ideal leverage of 0.5. Given the full symmetry of this model, the partial effects of three input parameters on five

Table 3. Comparative statics of Model I – no drift with symmetric penalty function and transaction costs

	Base case	Change in σ	Change in k	Change in q
Input:				
Standard deviation, σ	0.050	0.070		
Transaction cost, $k_a = k_b$, \$	0.050		0.030	
Penalty factor, q	2.000			1.500
Output:				
Lower debt barrier, a^*	0.361	0.335	0.378	0.350
Return point, r^*	0.500	0.500	0.500	0.500
Upper debt barrier, b^*	0.639	0.665	0.622	0.650
Leverage shift, $r^* - a^* = b^* - r^*$	0.139	0.165	0.122	0.150
Excess capital cost, z^*	0.013	0.018	0.010	0.011
Refinancing interval, v^* years	7.746	5.533	6.000	8.944

Assuming a firm with one dollar of assets, and an ideal leverage of 0.5.

Table 4. Comparative statics of asymmetric Models II, III and IV

	Model II		Model III		Model IV	
	(1)	(2)	(3)	(4)	(5)	(6)
Input:						
Standard deviation, σ	0.050	0.050	0.050	0.050	0.050	0.050
DRIFT, μ	0.000	0.000	0.000	0.000	-0.050	0.050
Transaction cost, k_a , \$	0.050	0.050	0.050	0.050	0.050	0.050
Transaction cost, k_b , \$	0.050	0.050	0.050	0.200	0.050	0.050
Ideal leverage, x_0	0.000	0.000	n/a	n/a	n/a	n/a
Penalty factor, q	1.000	1.100	0.000	0.000	0.000	0.000
a	0.000	0.000	0.400	0.400	0.400	0.400
b	endogen.	endogen.	0.600	0.600	0.600	0.600
Output:						
Lower debt barrier, a^*	0.000	0.000	0.400	0.400	0.400	0.400
Return point, r^*	0.079	0.077	0.500	0.467	0.548	0.452
Upper debt barrier, b^*	0.215	0.210	0.600	0.600	0.600	0.600
Leverage shift, $r^* - a^*$	0.079	0.077	0.100	0.067	0.148	0.052
Leverage shift, $b^* - r^*$	0.137	0.133	0.100	0.133	0.052	0.148
Excess capital cost, z^*	0.023	0.024	0.013	0.028	0.020	0.020
Refinancing interval, v^* years	4.309	4.108	4.000	3.556	2.461	2.461

Assuming a firm with one dollar of assets. Recall:

Model II: quadratic penalty function with a minimum $x_0 = 0$, a lower fixed barrier $a = 0$, and no drift.

Model III: zero penalty function, fixed upper and lower barriers, and no drift.

Model IV: zero penalty function, fixed upper and lower barriers, and non-zero drift.

Input variables subjected to a change are displayed in bold face.

optimal output parameters are displayed in four numerical columns including a base case and three parameter shifts subject to the condition $k_a = k_b$.

The base case consists of the input parameters $\sigma = 0.05$, $k_a = k_b = 0.05$, and $q = 2$ which generate the optimal output parameters $a^* = 0.361$, $r^* = 0.5$, $b^* = 0.639$, $z^* = 0.013$ and $v^* = 7.7$ years. The optimal barriers are placed at the same distance from the optimal return point, $r^* - a^* = b^* - r^* = 0.139$, implying the optimal leverage range $0.139 + 0.139 = 0.278$. The excess (minimum) cost of capital $z^* = 0.013$ is in addition to the unspecified minimum WACC fixed by the deterministic model underlying the penalty function.

Figures in the second numerical column reveal that an increase in σ from 0.05 to 0.07 causes an increase in the optimal leverage range from 0.278 to 0.330, which partially offsets the increase in periodic transaction costs. The excess capital cost increases from 0.013 to 0.018, the same increment as the full cost of capital, and the average refinancing interval decreases from 7.7 years to 5.5 years.

Results in the third column show that a decrease in the unit transaction cost from 0.05 to 0.03 causes a decrease in the leverage range from 0.278 to 0.244, which partially offsets the resulting decrease in periodic transaction costs. The excess capital costs decreases from 0.013 to 0.010, and the refinancing interval decreases from 7.7 years to 6.0 years.

The fourth column reveals how a decrease in the penalty factor from 2 to 1.5 causes an increase in the leverage range from 0.278 to 0.300. The excess capital cost decreases from 0.013 to 0.011, and the refinancing interval increases from 7.7 years to 8.9 years.

Models II–IV. Table 4 devotes two columns to each of these models, using bold face to distinguish input parameters subjected to a change.

Model II is symmetric except for its monotonically increasing quadratic penalty function with an ideal leverage at zero. Consistently, the leverage lower barrier is set exogenously at zero. An increase from 1.0 to 1.1 in the penalty slope coefficient causes a decrease in the upper barrier from 0.215 to 0.210, and a parallel decrease in the return point from 0.079 to 0.077. The excess cost of capital increases from 0.023 to 0.024, and the refinancing interval decreases from 4.3 to 4.1 years.

Model III assumes leverage indifference (no penalty function) with exogenous barriers set at 0.4 and 0.6. The full symmetry of column (3) is broken by an increase in the unit cost of stock issuance from $k_b = 0.05 = k_a$ to $k_b = 0.20 > k_a = 0.05$, causing a decrease in the return point from 0.5 (halfway between the barriers) to 0.467, an increase in the excess cost of capital from 0.013 to 0.028, and a decrease in the refinancing interval from 4.0 to 3.6 years.

Model IV maintains the assumptions of Model III of leverage indifference with exogenous barriers at 0.4 and 0.6. The full symmetry of column (3) is broken in Model IV by drift. By comparison to the scenario in column (3), negative drift of -0.05 in column (5) causes an increase in the return point from 0.5 to 0.548, an increase in the excess cost of capital from 0.013 to 0.020, and a decrease in the refinancing interval from 4.0 to 2.5 years. A Positive drift of 0.05 causes a

symmetric change in the return point, but the same changes in the excess cost of capital and the refinancing interval.

6. SUMMARY AND CONCLUSIONS

This study was motivated by the mounting evidence that the financing of firms in the real world is dominated by a hierarchy based on differential transaction costs, a behaviour which is not explained by traditional theories of static tradeoff. Static tradeoff theories, which do not explicitly treat the impact of transaction costs, do not explain the policy of asymmetry between frequent small debt transactions and infrequent large equity transactions. Nor do these theories explain why the debt ratio is allowed to wander a considerable distance from its alleged static optimum, or how much of a distance should be tolerated. We offer a class of diffusion models that mimic this behaviour in a stochastic-dynamic framework and are designed to optimize a financing strategy using any static tradeoff theory as input. The models developed reveal the determinants of the size and frequency of equity transactions and the range of values over which leverage variations are tolerated in four generic scenarios. They also yield a new formulation of the cost of capital which recognizes stochastic transaction costs and a penalty for deviation from any static-optimal leverage.

Our class of models augments the pecking order theory, provides a flexible quantitative framework for its implementation as a decision tool, and facilitates the formulation of additional hypotheses for its empirical validation. Symmetrically, our results show the importance of dynamic factors in designing and interpreting empirical tests of static tradeoff theories.

Our results have important implications for the role played by static tradeoff theories in a stochastic dynamic framework. One such implication is that the static-optimal leverage has no direct effect on the firm's leverage policy in this setting. The target leverage for refinancing transactions is different from the static-optimal leverage, and the mean leverage is generally different from both. As a consequence, the latter cannot be used to estimate the former. Another implication of our results is that even when the mean leverage is equal to the static optimum, mean reversion in the form of continuous partial leverage adjustment is not optimal behaviour and therefore not a legitimate test for the existence of a static tradeoff in a dynamic setting. Still another implication of our results is that the pecking order theory is consistent with static tradeoff theories and does not require the assumption of leverage indifference. Observed wide variations in the leverage ratio cannot be interpreted as evidence of leverage indifference.

APPENDIX: FEATURES OF OUR MODELS VS. THE MODELS OF FISCHER-HEINKEL-ZECHNER AND MAUER-TRIANTIS

BGY models	FHZ model	MT model
1. Leverage variable studied		
<ul style="list-style-type: none"> ● Unspecified, book-based, monotonically increasing in debt and decreasing in equity 	<ul style="list-style-type: none"> ● (Market assets)/(Book debt) 	<ul style="list-style-type: none"> ● Book debt
2. Source of leverage volatility		
<ul style="list-style-type: none"> ● Debt book value due to incremental borrowing and debt repayment 	<ul style="list-style-type: none"> ● Assets' market value 	<ul style="list-style-type: none"> ● Product price
3. External refinancing between barriers		
<ul style="list-style-type: none"> ● Debt 	<ul style="list-style-type: none"> ● None 	<ul style="list-style-type: none"> ● None
4. Transactions when hitting barriers		
<ul style="list-style-type: none"> ● Issuance or repurchase of incremental stock in lump sums offset by debt transactions 	<ul style="list-style-type: none"> ● Re-issuance of entire debt in the desired amount offset by stock transaction 	<ul style="list-style-type: none"> ● Issuance or repayment of incremental debt in lump sums offset by stock transaction
5. Refinancing costs at barriers		
<ul style="list-style-type: none"> ● Fixed per incremental equity transaction 	<ul style="list-style-type: none"> ● Semi-fixed semi-variable vis. incremental debt 	<ul style="list-style-type: none"> ● Fixed and proportional (linear) vis. incremental debt
6. Optimal return point(s)		
<ul style="list-style-type: none"> ● Single fixed 	<ul style="list-style-type: none"> ● Claimed single fixed – should be dual fixed (error in model) 	<ul style="list-style-type: none"> ● Dual changing over time
7. Analytical solutions for optimal values		
<ul style="list-style-type: none"> ● Two optimal barriers ● Optimal return point ● Optimal mean leverage ● Excess capital cost ● Expected time between transactions ● Various elasticities 	<ul style="list-style-type: none"> ● None 	<ul style="list-style-type: none"> ● None
8. Consistency with static theories		
<ul style="list-style-type: none"> ● Optimal leverage with quadratic penalty function (other functions possible) ● Leverage indifference 	<ul style="list-style-type: none"> ● Optimal leverage (with financial distress and US taxes) ● Possibly other optimal leverage ● Possibly leverage indifference 	<ul style="list-style-type: none"> ● Optimal leverage (with financial distress and US taxes) ● Possibly other optimal leverage ● Possibly leverage indifference
9. Relationship to pecking order behaviour		
<ul style="list-style-type: none"> ● Consistent 	<ul style="list-style-type: none"> ● Inconsistent 	<ul style="list-style-type: none"> ● Inconsistent

REFERENCES

- Asquith, P. and Mullins, D.W. (1986) Equity issues and offerings dilution, *Journal of Financial Economics*, **15**, 61–89.
- Auerbach, A.S. (1985) Real determinants of corporate leverage, in B.M. Friedman (ed.) *Corporate Capital Structure in the United States*, National Bureau of Economic Research, pp. 301–322.
- Bagley, C.N. and Yaari, U. (1996) Financial leverage strategy with transaction costs, *Applied Mathematical Finance*, **3**, 191–208.
- Bernanke, B.S. and Gertler, M. (1989) Agency costs, net worth, and business fluctuations, *American Economic Review*, **79**, 14–33.
- Bernanke, B.S. and Gertler, M. (1990) Financial fragility and economic performance, *Quarterly Journal of Economic*, **105**, 87–114.
- Bowen, R.M., Daley, L.A. and Huber, C.C. (1982) Evidence on the existence and determinants of inter-industry differences in leverage, *Financial Management*, **11**, 10–20.
- Cai, F. and Ghosh, A. (1995) Test of the capital structure theory: a binomial approach, Multinational Finance Conference, Philadelphia, June 20–22.
- Claggett, E.T. (1992) Capital structure: convergent and pecking order evidence, *Review of Financial Economics*, March, 35–48.
- Dann, L.Y. (1981) Common stock repurchases: an analysis of returns to bondholders and stockholders, *Journal of Financial Economics*, **9**, 113–138.
- Eckbo, B.E. (1986) Valuation effects of corporate debt offerings, *Journal of Financial Economics*, **15**, 119–152.
- Elton, E. and Gruber, M. (1968) The effects of share repurchase on the value of the firm, *Journal of Finance*, **23**, 135–149.
- Ferris, K.R., Melnik, A. and Rappaport, A. (1978) Factors influencing the pricing of stock repurchase, *Quarterly Review of Economics and Business*, **18**, 31–39.
- Fischer, E.O., Heinkel, R. and Zechner, J. (1989) Dynamic capital structure choice: theory and tests, *Journal of Finance*, **44**, 19–40.
- Gale, D. and Hellwig, M. (1985) Incentive-compatible debt contracts I: the one-period problem, *Review of Economic Studies*, **52**, 647–664.
- Iglehart, D.L. (1963) Optimality of (s,S) policies in infinite horizon dynamic inventory problem, *Management Science*, **9**, 259–267.
- Jalilvand, A. and Harris, R.S. (1984) Corporate behaviour in adjusting to capital structure and dividend targets: an econometric study, *Journal of Finance*, **39**, 127–145.
- Jensen, M.C. and Meckling, W. (1976) Theory of the firm: managerial behaviour, agency costs, and ownership structure, *Journal of Financial Economics*, **3**, 305–360.
- Karlin, S. and Taylor, H.M. (1981) *A Second Course in Stochastic Processes*. New York: Academic Press.
- Lev, B. (1969) Industry averages as targets for financial ratios, *Journal of Accounting Research*, Autumn, 290–299.
- Marsh, P. (1982) The choice between equity and debt: an empirical study, *Journal of Finance*, **37**, 121–144.
- Masulis, R.W. (1980) Stock repurchase by tender offer: an analysis of the causes of common stock price changes, *Journal of Finance*, **35**, 305–319.
- Masulis, R.W. and Korwar, A.N. (1986) Seasoned equity offerings: an empirical investigation, *Journal of Financial Economics*, **15**, 91–118.
- Mauer, D.C. and Triantis, A.J. (1994) Interactions of corporate financing and investment decisions: a dynamic framework, *Journal of Finance*, **49**, 1253–1277.

- Mikkelson, W.H., and Partch, M.M. (1986) Valuation effects of security offerings and the issuance process, *Journal of Financial Economics*, **15**, 31–60.
- Miller, M.H. (1977) Debt and taxes, *Journal of Finance*, **32**, 261–275.
- Miller, M.H. and Orr, D. (1966) A model for the demand for money by firms, *Quarterly Journal of Economics*, **80**, 413–435.
- Miller, M.H. and Orr, D. (1968) The demand for money by firms: extensions of analytic results, *Journal of Finance*, **23**, 735–759.
- Modigliani, F. and Miller, M.H. (1958) The cost of capital, corporation finance and the theory of investment, *American Economic Review*, **48**, 261–297.
- Myers, S.C. (1984a) The capital structure puzzle, *Journal of Finance*, **39**, 575–592.
- Myers, S.C. (1984b) The search for optimal capital structure, *Midland Corporate Finance Journal*, **1**, 6–16.
- Myers, S.C. and Majluf, N. (1984) Corporate financing and investment decisions when firms have information investors do not have, *Journal of Financial Economics*, **13**, 187–221.
- Scarf, H. (1960) The optimality of (S, s) policies in the dynamic inventory problem, *Mathematical Methods in Social Sciences*. Stanford: Stanford University Press.
- Shyam-Sunder, L. and Myers, S.C. (1992) Testing Static Trade-off Against Pecking Order Models of Capital Structure. Presented at the Inaugural International Conference on Financial Management, Suffolk University, Boston, MA.
- Smith, C. (1977) Alternative methods for raising capital: rights versus underwritten offerings, *Journal of Financial Economics*, **5**, 273–307.
- Smith, C. (1986) Investment banking and the capital acquisition process, *Journal of Financial Economics*, **15**, 3–29.
- Stonehill, A., Beekhuisen, T., Write, R., Remmers, L., Toy, N., Pares, A., Shapiro, A., Egan, D. and Bates, T. (1975) Financial goals and debt ratio determinants: a survey of practice in five countries, *Financial Management*, **3**, 27–47.
- Taggart, R.A. (1977) A model of corporate financing decisions, *Journal of Finance*, **32**, 1467–1484.
- Titman, S. and Wessels, R. (1988) Determinates of capital structure choice, *Journal of Finance*, **43**, 1–19.
- Townsend, R. (1979) Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory*, **21**, 265–293.
- Vermaelen, T. (1981) Common stock repurchases and market signaling: an empirical analysis, *Journal of Financial Economics*, **9**, 139–183.